Math 251 Practice Exam 3 Name_____

K. Pericak-Spector

YOU MUST SHOW ALL WORK!

[25] 1. Consider the following curve:

$$\mathbf{r}(t) = \langle e^t \sin t, e^t \cos t, e^t \rangle$$

Find the

a) velocity

b) $|\mathbf{r}'|$ (This is the speed.)

c) unit tangent vector

d) length of the curve for $1 \leq t \leq 3$

e) equation of tangent line at $(0, 1, 1)$.

[35] 2. a) Determine the point of intersection of the two lines $\begin{aligned} x &= 5+2t \\ y &= 4+3t \\ z &= -1-t \end{aligned}$ $\begin{aligned} x &= -1+s \\ y &= -4+s \\ z &= -7+4s \end{aligned}$.

b) Is the intersection perpendicular?

c) Find the equation of the plane which contains both lines.

d) Find the equation of the line where this plane hits the z axis.

3. Consider $\mathbf{r} = \langle 2t^3, 3t^2, 4\rangle$.

[15] a) Compute the curvature.

[5] b) Find the radius.

[5] c) Graph the radius as a function of t. (It is easier to see what is happening if you simplify the expression.)

[5] 4. Consider the curve represented by $\mathbf{r} = (e^{-t}\sin t)\mathbf{i} + (e^{-t}\cos t)\mathbf{j}$. In this case, you cannot eliminate the parameter but you can still sketch the curve in the $xy-$plane and give the orientation of the graph. (Hint: What is $\mathbf{r}(0), \mathbf{r}\left(\frac{\pi}{2}\right), \mathbf{r}(\pi), \mathbf{r}(2\pi)$?) Sketch the graph.

[10] 5. Match the equation to its graph. Put the number by the letter.

a) _____ $z = r^2$

b) _____ $y = x^2 + z^2$

c) _____ $z = x^2 + y^2$

d) _____ $\rho = 3$

e) _____ $z^2 + 1 = x^2 + y^2$

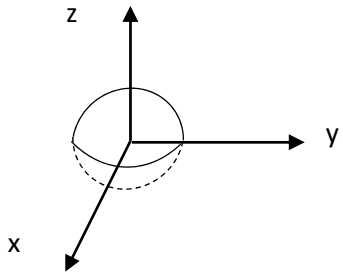
f) _____ $\rho = \cot\varphi \csc\varphi$

g) _____ $r^2 + z^2 = 9$

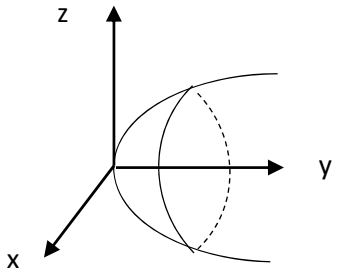
h) _____ $x = y^2 + z^2$

i) _____ $z^2 + 1 = r^2$

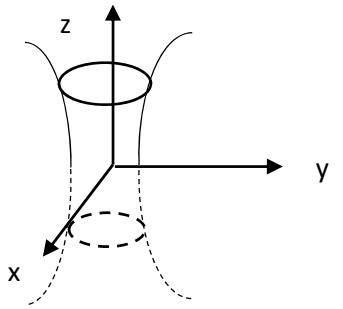
A



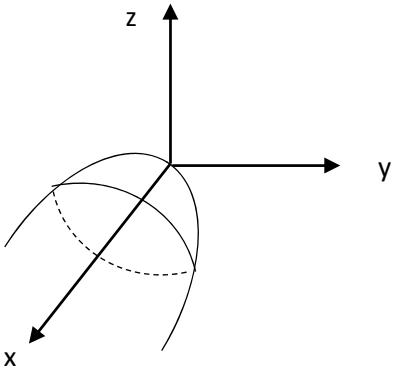
B



C



D



E