Summary of Velocity, Acceleration, and Curvature

Let *C* be a curve (in the plane or in space) given by the position function

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$$

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}.$$

Velocity vector, speed, and acceleration vector:

Velocity: $\mathbf{v}(t) = \mathbf{r}'(t)$

Speed: $\|\mathbf{v}(t)\| = \dfrac{ds}{dt} = \|\mathbf{r}'(t)\|$

Acceleration: $\mathbf{a}(t) = \mathbf{r}''(t) = a_{\mathbf{T}}\mathbf{T}(t) + a_{\mathbf{N}}\mathbf{N}(t)$

Unit tangent vector:

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

Principal unit normal vector:

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

Components of acceleration:

$$a_{\mathbf{T}} = \mathbf{a} \bullet \mathbf{T} = \frac{\mathbf{v} \bullet \mathbf{a}}{\|\mathbf{v}\|}$$

$$a_{\mathbf{N}} = \mathbf{a} \bullet \mathbf{N} = \sqrt{\|\mathbf{a}\|^2 - {a_{\mathbf{T}}}^2} = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|}$$

Formulas for curvature in the plane or in space:

$$k = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

$$k = \frac{\mathbf{a}(t) \bullet \mathbf{N}(t)}{\|\mathbf{v}(t)\|^2}$$