Math 305 **Homogeneous Equations**

A first order homogeneous equation can be written in the form

$$y' = f\left(\frac{y}{t}\right)$$

Basically if y appears then it needs to be divided by t.

Steps:

1. Make sure the equation is homogeneous.

2. Let $v(t) = \dfrac{y(t)}{t}$.

3. Rewrite $y(t) = tv(t)$, differentiate with respect to $t : \dfrac{dy}{dt} = v + t\dfrac{dv}{dt}$.

4. Substitute the above in for $\dfrac{dy}{dt}$, rewrite all $\dfrac{y}{t}$ as v.

5. Equation is now separable.

Example A. $ty' = y + t\cos\dfrac{y}{t}$

1. Notice the equation can be rewritten as $y' = \dfrac{y}{t} + \cos\dfrac{y}{t}$.

2. $v(t) = \dfrac{y(t)}{t}$

3. $y(t) = tv(t)$ so that $\dfrac{dy}{dt} = v + t\dfrac{dv}{dt}$

4. $v + t\dfrac{dv}{dt} = v + \cos v$

5. $t\dfrac{dv}{dt} = \cos v$ Now it's separable. Can you finish the problem?

Example B. $y' = \dfrac{t-y}{t+y}$

1. Notice equation can be written as $y' = \dfrac{1 - \frac{y}{t}}{1 + \frac{y}{t}}$.

2. $v(t) = \dfrac{y(t)}{t}$

3. $y(t) = tv(t)$ so that $\dfrac{dy}{dt} = v + t\dfrac{dv}{dt}$

4. $v + t\dfrac{dv}{dt} = \dfrac{1-v}{1+v}$

5. $t\dfrac{dv}{dt} = \dfrac{1-2v-v^2}{1+v}$

Problems:

1. $ty' - y = te^{y/t} \qquad y(1) = 0$

2. $y' = \dfrac{t+y}{t-2y} \qquad y(1) = 3$

3. $y' = \dfrac{4t+3y}{3t+y} \qquad y(0) = 1$

4. $ty' - y = \dfrac{t^2+ty}{t-y} \qquad y(1) = 2$

5. $y' = \dfrac{yt^2 - 8t^3}{t^3+y^3} \qquad y(1) = 3$

Solutions:

1. $e^{-y/t} = -\ln t + 1$

2. $\dfrac{1}{\sqrt{2}} \tan^{-1} \dfrac{\sqrt{2}y}{t} = -\ln t + \dfrac{1}{\sqrt{2}} \tan^{-1} 3\sqrt{2} - \ln 10$

3. $(2t+y) = -(2t-y)^5$

4. $-\dfrac{y}{t} + 2\ln\left(1 + \dfrac{y}{t}\right) = \ln t - 2 + \ln 3$

5. Work in progress.