Math 251 Practice Problems
K. Pericak-Spector Chapter 11

1. Let $f(x,y) = x^2 + y^2$ and $x = r\cos\theta$ and $y = \sin\theta$. Compute f_θ and f_r.

2. Let $f(x,y) = \arctan(y/x)$. Let $x = e^t$ and $y = t^2$. Compute df/dt.

3. Find the directional derivative of $f(x,y) = e^x \sin y$ in the direction $\mathbf{v} = \langle 1, 1\rangle$.

4. Find the gradient of the function $w = 1/(1 - x^2 - y^2 - z^2)^{1/2}$ and the maximum value of the directional derivative at $\left(\frac{1}{2}, 0, \frac{1}{2}\right)$.

5. Find an equation of the tangent plane to the surface $h(x,y) = \ln(x^2 + y^2)^{1/2}$ at $(3, 4, \ln 5)$.

6. Find a unit normal vector to the surface $x^2 + y + z^2 = 9$ at $(2, -1, 2)$.

7. Find an equation of the tangent plane and equations of the normal line to $xyz = 10$ at $(1, 2, 5)$.

8. Find the absolute extrema of the function $f(x,y) = x^2 - 4xy$ over the region $R = \{(x,y) : 0 \leq x \leq 4, 0 \leq y \leq \sqrt{x}\}$.

9. Identify any extrema of the function $f(x,y) = x^3 - 3xy + y^3$.

10. Use Lagrange multipliers to minimize and miximize (if possible) each of the funcitons f subject to the indicated constraint.

a) $f(x, y) = x^2 y, \qquad 2x^2 + y^2 = 2$

b) $f(x, y) = x^2 + y^2, \qquad x^2 - 2x + y^2 + 2y = -1$

c) $f(x, y) = x^4 - x^2 y^2, \qquad x^2 + y^2 = 1$

d) $f(x, y, z) = x^2 + y^2 + z^2, \qquad z^2 = 4xy + 1$

e) $f(x, y, z) = xyz, \qquad x^2 + 4y^2 + 2z^2 = 8$

f) $f(x, y, z) = x + 2x - 3z, \qquad z = 4x^2 + y^2$

11. Minimize and maximize each of the functions f subject to the indicated constraint.

a) $f(x, y) = x^4 - x^2 y^2, \qquad x^2 + y^2 \leq 1$

b) $f(x, y) = x^2 y, \qquad 2x^2 + y^2 \leq 2$

c) $f(x, y, z) = xyz, \qquad x^2 + 4y^2 + 2z^2 \leq 8$

d) $f(x, y) = x^4 - 4x^2 y^2 + y^4, \qquad x^2 + y^2 \leq 9$

e) $f(x, y) = 4xy - x^3 y - y^3 x, \qquad x^2 + y^2 \leq a^2$