Math 251 Practice Exam 2 Name________________

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1. a) Let $f(x,y,z) = x\tan^{-1}\left(\frac{y}{z}\right)$. Find the directional derivative of f at the point $(-3,1,1)$ in the direction $\langle 10, 11, -2\rangle$.

   b) What is the **rate** of maximum increase at this point? (Not the direction!)

2. Use the chain rule to compute the partial derivative $\frac{df}{dt}$ at the point $t = 3$.

$$f(x,y) = x^2 e^{xy}$$

$$x = \sin t \quad y = t^2 + t$$

3. a) Find the equation of the tangent plane to the surface at the point $(1,-1,4)$ to $z^2 - 2x^2 - 2y^2 = 12$.

   b) Find the unit vector perpendicular to the tangent plant to the above surface at this point.

   c) The tangent plane intersects the xy plane to form a line. Find the equation of this line.

4. Find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y + 4z = 49$.

5. Find all critical points of the following function and use the second derivative test to determine which are relative maxima, relative minima, and saddle points.

$$f(x,y) = \frac{x^2y^2 - 8x + y}{xy}$$

6. Find the absolute extrema of the function $f(x,y) = 3x^2 + 2y^2 - 4y$ on the region bounded by $y = x^2$ and $y = 12$.