Math 251 Exam 2 Name________________________

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[10] 1. Let $f(x,y) = x^2 - 2xy^3$. Find the directional derivative of f at the point $(3,-1)$ in the direction $\langle 3,4\rangle$.

[15] 2. Find the maximum and minimum value of $f(x,y) = x^2 y$ subject to the constraint $4x^2 + 9y^2 = 36$.

[20] 3. a) Find an equation of the tangent plane at $P(0, 3, -1)$ to the surface with equation $ze^x + e^{z+1} = xy + y - 3$.

b) The tangent plane intersects the xz plane to form a line. Find the equation of this line.

[10] 4. Use the chain rule to compute the partial derivative $\dfrac{\partial f}{\partial t}$ at $(t, s) = (-1, -1)$ where $f(x, y, z) = x^3 + yz^2, \qquad x = t^2 + x, y = t + x^2, \qquad z = st.$

[25] 5. Find the absolute extrema of the function $f(x, y) = x^3 + x^2y + 2y^2$ on the region bounded by $0 \leq x + y \leq 1$ and $x, y \geq 0$.

[20] 6. Find all critical points of the following function and use the second derivative test to determine which are relative maxima, relative minima, and saddle points.

$$f(x, y) = e^{x+y} - xe^{2y}$$

[10] 7. A bug located at $(3, 9, 4)$ begins walking in a straight line toward $(5, 7, 3)$. At what rate is the bug's temperature changing if the temperature is $T(x, y, z) = xe^{y-z}$? Units are measured in meters and degrees Celsius.