We need to evaluate $\int_R\int f(x,y)dA$, given a region R in the x-y plane. Note that to set up the integral only the region is important, not the integrand.

1. Draw the region R in the x-y plane.

2. Find points of intersection of the curves.

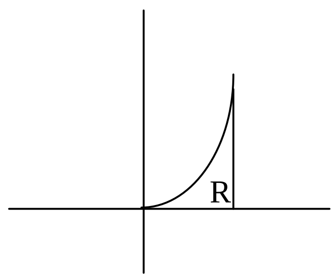
3. If integrating with respect to y first, fix x and ask yourself how does y vary (lower curve to upper curve) (limits will involve x's or constants).

4. How does x-vary to pick up every point in region (left to right) (limits will involve constants).

Note: If integrating with respect to x first, fix y and ask yourself how does x vary (left curve to right curve) (limits will involve y's or constants). Then ask yourself how does y-vary to pick up every point in region (lower to upper) (limits will involve constants).

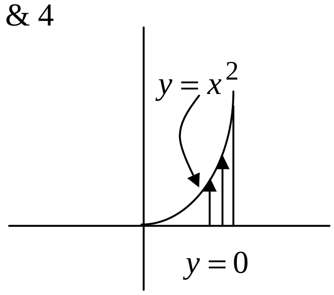
Example 1 R: region bounded by $y = x^2, y = 0, x = 2$.

1 & 2



Points of intersection:
$y = x^2$ and $x = 2$ hit at (2,4)
$y = x^2$ and $y = 0$ hit at (0,0)

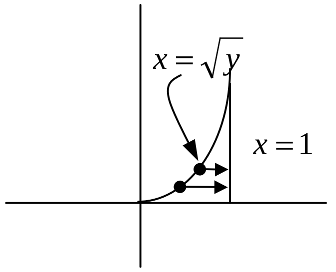
3 & 4



For each x,y starts at y=0 and ends at $y = x^2$.
To pick up all x, x starts at $x = 0$ and ends at $x = 2$.

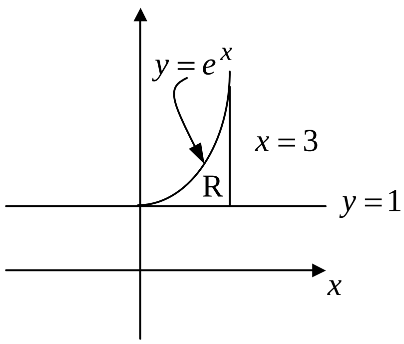
Thus $\int_0^2\int_0^{x^2} dy\ dx$.

To integrate with respect to *x* first, we need to write *x* as a function of *y*. Note that if $y = x^2 \Rightarrow x = \sqrt{y}$ (want + square root since you are in first quadrant).

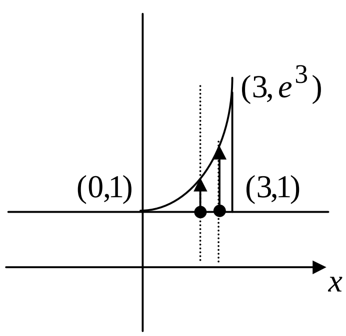


For each *y*, *x* starts at $x = \sqrt{y}$ and ends at $x = 2$.
To pick up all *y*, *y* starts at $y = 0$ and ends at $y = 4$.
Thus $\int_0^4 \int_{\sqrt{y}}^2 \, dx \; dy$.

Example 2 $R: \; y = 1 \quad y = e^x \quad x = 3$

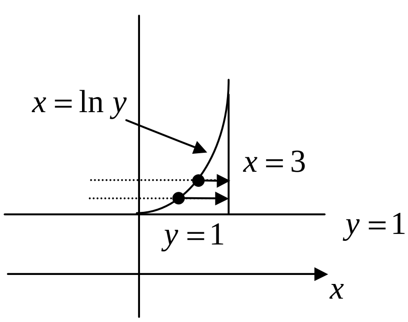


Points of intersection:
$y = e^x$ hits $y = 1$ at $x = 0$
$y = e^x$ hits $x = 3$ at $y = e^3$



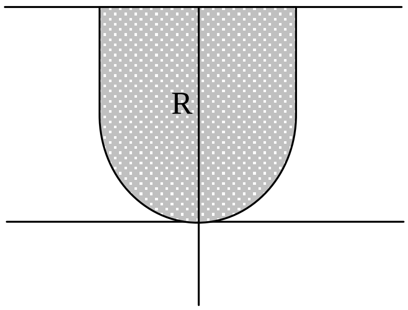
For each *x*, *y* goes from $y = 1$ to $y = e^x$.
To pick up all *x*, *x* starts at $x = 0$ and ends at $x = 3$.
Thus $\int_0^3 \int_1^{e^x} dy \, dx$.

To integrate with respect to *x* first we need to write *x* as a function of *y*. Note if $y = e^x \Rightarrow x = \ln y$.

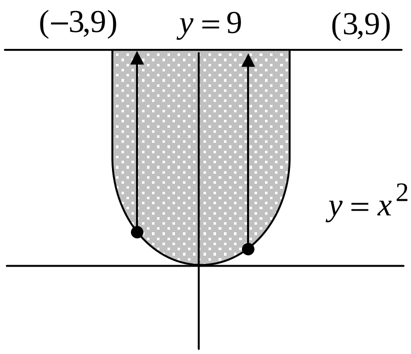


For each *y*, *x* starts at $x = \ln y$ and ends at $x = 3$.
To pick up all *y*, *y* starts at $y = 1$ and ends at $y = e^3$.

Example 3 $R: y = x^2$ and $y = 9$

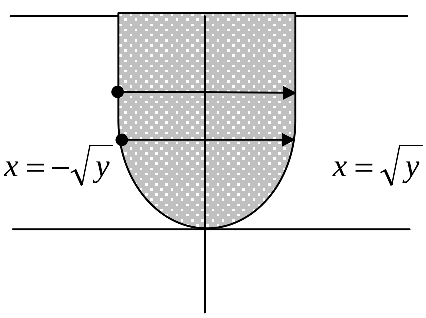


Points of intersection:
The curves hit when $x^2 = 9 \Rightarrow x = \pm 3$.



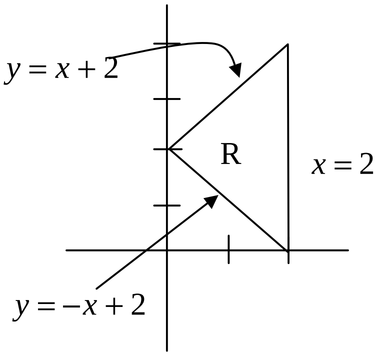
For each *x*, *y* starts at the lower curve $y = x^2$ and ends at the upper curve $y = 9$. To pick up all *x*, *x* starts at the left-hand point $x = -3$ and ends at the right-hand point $x = 3$. Thus $\int_{-3}^{3}\int_{x^2}^{9} dy\,dx$.

To integrate with respect to *x* first we need to write *x* as a function of *y*. The left-hand curve can be written as $x = -\sqrt{y}$ and the right-hand curve can be written as $x = \sqrt{y}$.

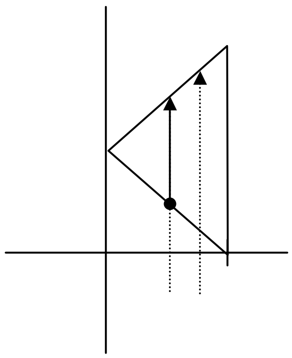


For each *y*, *x* goes from $x = -\sqrt{y}$ to $x = +\sqrt{y}$. To pick up all *y*, *y* starts at $y = 0$ and ends at $y = 9$.
Thus $\int_{0}^{9}\int_{-\sqrt{y}}^{\sqrt{y}} dx\ dy$.

Example 4 R bounded by $y = x + 2$, $y = -x + 2$, $x = 2$.

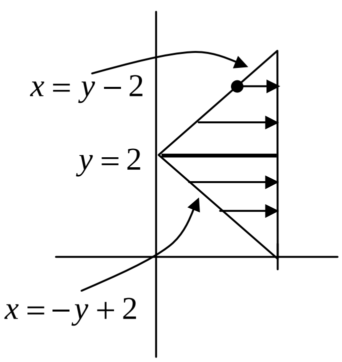


Points of intersection:
$y = x + 2$ hits $x = 2$ at (2,4)
$y = -x + 2$ hits $x = 2$ at (2,0)
$y = x + 2$ hits $y = -x + 2$ at (0,2)



Find x then y starts at the curve $y = -x + 2$ and ends at the curve $y = x + 2$. To get all values of x start at $x = 0$ and end at $x = 2$.

Once again solve both equations for x. Thus $x = y - 2$ and $x = -y + 2$.



Now fix y, then x goes from the left curve to the right curve. the problem is that there are two "left" curves. So when $y > 2$ we have x going from $x = y - 2$ to $x = 2$ when $y < 2$ we have x going from $x = -y + 2$ to $x = 2$. Thus we will have two integrals. $\int_2^4 \int_{y-2}^2 dx\, dy + \int_0^2 \int_{-y+2}^2 dx\, dy$.

Problems Sketch the region and set up the integral.

1. $\iint_R 16x\,dA$ $\quad R: y = 9 - 4x^2 \quad y = 0 \quad x = 0,\ x = \frac{3}{2}$

2. $\iint_R dA$ $\quad R: x = e^y,\ x = 2,\ y = 0$

3. $\iint_R dA$ $\quad R:\ x - \text{axis},\ y - \text{axis},\ y = x + 2$

4. $\iint_R 6x\,dA$ $\quad R: x^2 + y^2 = 4$

Reverse order of integration and evaluate.

5. $\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos\left(16\pi x^5\right) dx\,dy$

6. $\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dx\,dy$

7. $\int_0^2\int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx$

8. $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\,dy\,dx$