Math 251 Practice Exam 3
K. Pericak-Spector

1. Find the area of the surface given by $f(x,y) = \sqrt{x^2+y^2}$ over the region given by $R = \{(x,y) : x^2+y^2 \leq 1\}$.

2. Consider $\int\int_{\mathrm{R}} \cos\frac{x}{y} d\mathrm{A}$ where R=$\{(x,y) : 1 \leq y \leq 2, y \leq x \leq y^3\}$.

   i.e. The region bounded by $y=1,\ y=2,\ y=x$, and $x=y^3$.

   a) Draw the region R in the $x-y$ plane.

   b) Set up the integral.

   c) Evaluate.

3. Evaluate the iterated integral.

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} x^2\, dy\, dx$$

   Hint: Covert to polar coordinates.

   a) Draw the region in the $x-y$ plane.

   b) Convert to polar coordinates.

   c) Evaluate.

4. Evaluate $\int\int\int_{\mathrm{V}} z^2 d\mathrm{V}$ where V is the solid that lies above the cone $\sqrt{3}z = \sqrt{x^2+y^2}$ and below by the sphere given by $x^2+y^2+z^2 = 25$. Hint: Use spherical coordinates.
   BONUS: Set up but do not evaluate, the integral in cylindrical coordinates.

5. Evaluate the integral.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{9} \sqrt{x^2+y^2} dz\, dy\, dx$$

   Hint: Use cylindrical coordinates. BONUS: Set up but do not evaluate, the integral in spherical coordinates.