Math 250 Review of Series Limit Comparison Test

Consider $\sum^{\infty} a_n$ and $\sum b_n$.

If $a_n, b_n \geq 0$ and $\lim_{h\to\infty} \frac{a_n}{b_n} = L$ where $L \neq 0$ or $L \neq \infty$. Then the series do the same thing.
You use the idea of p-series with this.

Note: If $p > 1$ then $\sum \frac{1}{n^p}$ converges. If $p \leq 1$ then $\sum_1^{\infty} \frac{1}{n^p}$ diverges.

Example:

1. $\sum_1^{\infty} \frac{1}{n^2+1}$ compare to $\sum \frac{1}{n^2}$ $\lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{1}{n^2+1}} = \lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$ $1 \neq 0$ $1 \neq \infty$

   since $\sum \frac{1}{n^2}$ converges p-series $p = 2 > 1 \Rightarrow \sum \frac{1}{n^2+1}$ converges

2. $\sum_1^{\infty} \frac{1}{n^2+1}$ compare to $\sum \frac{1}{n}$ $\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{n}{n^3+1}} = \lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$ $1 \neq 0$ $1 \neq \infty$

   since $\sum \frac{1}{n}$ diverges p-series $p = 1 \leq 1 \Rightarrow \sum \frac{n}{n^2+1}$ diverges

Determine if the following series converge or diverge.

1. $\sum_1^{\infty} \frac{n}{n^3+1}$
2. $\sum_1^{\infty} \frac{n+1}{n^2+3n+5}$
3. $\sum_1^{\infty} \frac{n+1}{\sqrt{n^3+1}}$
4. $\sum_1^{\infty} \frac{1}{n^{1/2}+1}$
5. $\sum_1^{\infty} \frac{n^2+3}{n^5+4}$

Hint 1: Works great when series only involves $\frac{\text{poly}}{\text{poly}}$ or roots of these.

Hint 2: You need to make an intelligent guess. If you over or under estimate your guess $L = 0$ or $L = \infty$ you need to make a better guess.