

K.A. Pericak-Spector

16.4 deals with line integrals in differential form and the curve C is closed. The idea is that parameterizing may be difficult and sometimes the integration is difficult as well (see second example.)

Evaluate $\int_C Pdx + Qdy$ for the given curves.

Example 1. $\int_C (y - x)dx + (2x - y)dy$

C : boundary of the region lying between the graphs of $y = x$ and $y = x^2 - x$.

Step 1. Draw region.



Step 2. Is it closed? Here, yes.

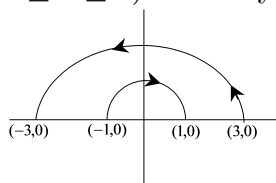
Step 3. Identify P and Q ; here $P = y - x$, $Q = 2x - y$.

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2 - 1 = 1$

Step 5. Evaluate. Here $\int_R \int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$. $\int_0^2 \int_{x^2-x}^x 1 dy dx = \int_0^2 (x - (x^2 - x)) dx = \int_0^2 (2x - x^2) dx = x^2 - \frac{x^3}{3} = 4 - \frac{8}{3} = \frac{4}{3}$.

Example 2: Compute $\int_C (\arctan x + y^2)dx + (e^y - x^2)dy$

C : bounded by the curves $y = 0$ ($-3 \leq x \leq -1$, $1 \leq x \leq 3$) above by $x^2 + y^2 = 1$ below by $x^2 + y^2 = 9$. Step 1. Draw region.



Step 2. Is it closed? Here, yes.

Step 3. Identify P and Q . Here $P = \arctan x + y^2$ $Q = e^y - x^2$.

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -2x - (2y)$.

Step 5. Evaluate. Here $\int_R \int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_R \int -2(x + y) dA$

It is easier to do this problem in polar coordinates.
$$\int_0^\pi \int_1^3 -2(\mathbf{r} \cos \theta + \mathbf{r} \sin \theta) \mathbf{r} d\mathbf{r} d\theta =$$

$$-2 \int_0^\pi \int_1^3 \mathbf{r}^2 (\cos \theta + \sin \theta) d\mathbf{r} d\theta = -2 \left[9 - \frac{1}{3} \right] (2)$$

A. $\int_C x e^y dx + e^x dy$ C: boundary of the region lying between the graph of $y = x$ and $y = x^2$.

Step 1. Draw region. *See Graph file.

Step 2. Is it closed? If yes, proceed.

Step 3. Identify P and Q .

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Step 5. Evaluate $\int_R \int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$.

B. $\int_C 2xy dx + (x + y) dy$ C: boundary of the region lying between the graphs of $y = 0$ and $y = 4 - x^2$.

Step 1. Draw region.

Step 2. Is it closed? If yes, proceed.

Step 3. Identify P and Q .

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Step 5. Evaluate $\int_R \int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$.

$$C. \int_C 2 \tan \frac{y}{x} dx + \ln(x^2 + y^2) dy \quad C : x = 4 + 2 \cos \theta \quad y = 4 + \sin \theta$$

Step 1. Draw region.

Step 2. Is it closed? If yes, proceed.

Step 3. Identify P and Q .

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Step 5. Evaluate $\int_R \int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$.