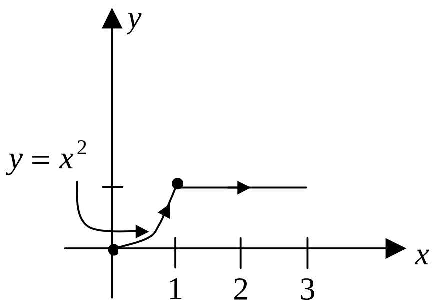
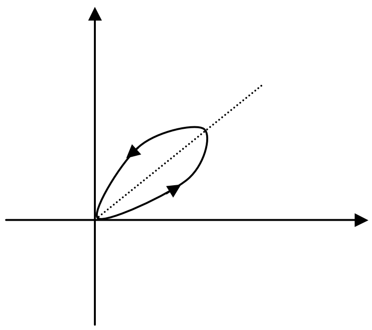
Math 251 Exam 4 Name____________________

Pericak-Spector

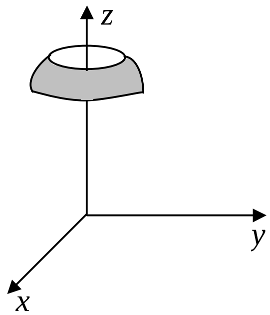
[20] 1. Compute the line integral $\int_C x\, ds$ along



[20] 2. Compute the line integral $\oint_C \left(e^{x^2} \sin x^3 - y\right)dx + \left(\frac{y^3}{y^5 + y^2} + x\right)dy$ where C is the boundary of one leaf of the clover rotated counterclockwise in the first quadrant given by $r = \sin 2\theta$.

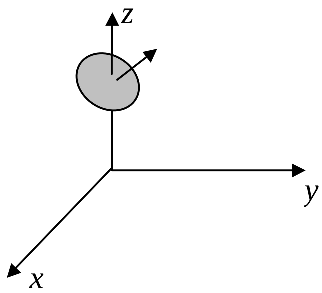


[20] 3. Compute the surface integral $\iint_S z^{-1} dS$ where S is the portion of the sphere $x^2 + y^2 + z^2 = 9$ where $1 \le x^2 + y^2 \le 4$ and $z \ge 0$.

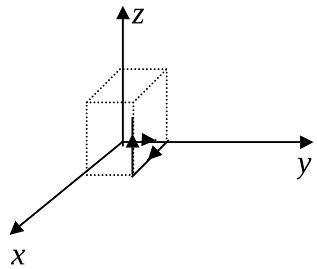


[20] 4. Compute the surface integral $\iint_S \mathbf{F}\cdot\mathbf{N}\ dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. $\mathbf{N}$ is the outward unit normal to S, and $\mathbf{F}$ is the vector field $\mathbf{F} = \langle xy^2, yz^2, zx^2 \rangle$.

[20] 5. Compute the surface integral $\iint_S \text{curl } \mathbf{H} \cdot \mathbf{N} \, dS$ where S is the portion of the surface of the plane $x + 2y + z = 16$ where $x^2 + y^2 \le 4$, the z component of **N** points in the $\langle 0,0,1 \rangle$ direction, H is the vector field $H = \langle y, -x, x + y \rangle$



[10] 6. Compute $\int_C (3x^2 + y)dx + (x + 3y^2 + z)dy + (y + 3z^3)dz$ where C starts at (0,0,0) and goes along the y-axis to (0,2,0) and then goes parallel to x-axis until it gets to (2, 2, 0) and then goes parallel to z-axis until it gets to (2, 2, 2). Hint: Conserve your energy.



Math 251 Exam 4 Answers

K. Pericak-Spector

1. $4 + \frac{1}{12}\left(5^{2/3} - 1\right)$

2. 1

3. $3\pi \ln \frac{8}{5}$

4. $\frac{(4\pi)}{5}(243)\left(\frac{4\pi}{5}\right)$

5. -8π

6. 32