Math 251 Final Exam Name_____
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[20] 1. a) Determine the equation of the line formed by the intersection of the planes

$$2x + y + 3z = 6$$
$$x - y + z = 2$$

b) Do the planes intersect perpendicularly? Explain your answer.

[20] 2. a) Find an equation of the tangent plane to the curve

$$y + y^2 - xyz = 1 \text{ at the point } (1, -1, 1)$$

b) Find the equation of the normal line at this point.

[30] 3. Consider the following curve: $\mathbf{r}(t) = \left\langle \frac{t^2}{2}, t^2, t^3 + 3 \right\rangle$ Find:

a) velocity

b) acceleration

c) length of curve from $-1 \leq t \leq 1$

d) curvature

[20] 4. a) Find the directional derivative of $f(x, y) = \dfrac{x}{x^2 + y^2}$ at $(1, 2)$ in the directional $\langle 3, 5 \rangle$.

b) What is the max/min rate of increase?

[20] 5. Find all critical points of the following function and use the second derivative test to determine which are relative maxima, relative minima, and saddle points. $f = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

[20] 6. Find the maximum and minimum values of $f = 2x+2y+z$ subject to the constraint $x^2+y^2+z^2 = 9$.

[20] 7. Evaluate $\int_0^1 \int_{3y}^3 \sin x^2 dxdy$.

[20] 8. Compute the line integral

$$\oint (y - \cos y)dx + (x \sin y)dy$$

where C is the circle $(x-3)^2 + (y+4)^2 = 4$ oriented counterclockwise.

[20] 9. Compute the surface integral

$$\int\int_S \text{curl } \langle 0, x, xz \rangle \cdot \mathbf{N} dS$$

where S is the spherical cap $x^2 + y^2 + z^2 \leq 1$ where $z \geq \frac{1}{2}$ and $\mathbf{N}$ points in the $\langle 0, 0, 1 \rangle$ direction.

[20] 10. Compute the surface integral $\int_S \langle x, y^2, z+y\rangle \cdot \mathbf{N} dS$ where S is the surface of the solid contained in the cylinder $x^2+y^2=4$ between the plane $z=x$ and $z=8$.