Math 251 – Final Exam Sheet

Rectangular Coordinates	(x, y, z)	Polar Coordinates (r, θ)	$x = r\cos\theta$ $y = r\sin\theta$
Cylinder Coordinates (r, θ, z)	$x = r\cos\theta$ $y = r\sin\theta$ $z = z$	Spherical Coordinates (ρ, φ, θ)	$x = \rho\sin\varphi\cos\theta$ $y = \rho\sin\varphi\sin\theta$ $z = \rho\cos\varphi$

	dA	dV
Rectangular	$dxdy$	$dxdydz$
Polar	$rdrd\theta$	
Cylindrical		$rdzdrd\theta$
Spherical		$\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

Let $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$.

Velocity $\mathbf{v}(t) = \dfrac{d\mathbf{r}}{dt}$

Acceleration $\mathbf{a}(t) = \dfrac{d\mathbf{v}}{dt}$

Unit tangent vector $\mathbf{T}(t) = \dfrac{\mathbf{v}(t)}{|\mathbf{v}(t)|}$

Curvature $\kappa = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$

Principal unit normal vector $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$

Binormal vector $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

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K. Pericak-Spector

[20] 1. a) Find the line of intersection of the two planes

$$x + y + 3z = 4$$
$$x - y + 7z = 14$$

b) Is this intersection perpendicular?

[20] 2. a) Find the directional derivative of $f(x, y) = e^{2x}y^2 - x^2y^3$ at the point $(2, 4)$ in the direction of the point $(-1, 2)$.

b) What is the direction of the maximum rate of increase?

[30] 3. Consider the following curve: $\mathbf{r}(t) = \left\langle e^t,\ e^{-t}, t\sqrt{2} \right\rangle$ Find:

a) velocity

b) acceleration

c) length of curve from $0 \leq t \leq \ln 2$

d) curvature

[20] 4. Find an equation of the tangent plane to the surface at $P(1, 2, -3)$ to

$$3x^2y - yz^2 = -12$$

[20] 5. Find the maximum and minimum for the function $f(x, y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 13$.

[20] 6. Find all critical points of the following function and use the second derivative test to determine which are relative maxima, relative minima, and saddle points. $f(x, y) = (x^2 - 2x)(2y - y^2) - 6y$

[20] 7. Evaluate $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dxdy$.

[20] 8. Compute the line integral

$$\int (y^2 - \ln(x^2 + y^2))dx + 2\arctan\frac{y}{x}dy$$

where C is the circle $x^2 + y^2 = 2y$ oriented counterclockwise.

[20] 9. Compute the surface integral $\int\int_S \mathbf{F}\cdot\mathbf{N}dS$ where S is the surface of the solid below the sphere $z=\sqrt{1-x^2-y^2}$ and above the cone $z=-1+\sqrt{x^2+y^2}$.

$$\text{Notice}\quad -1+\sqrt{x^2+y^2}\le z\le\sqrt{1-x^2-y^2}.$$

$\mathbf{N}$ is the outward unit normal to S, and $\mathbf{F}$ is the vector field

$$\mathbf{F}=\langle x-xy, y, zy\rangle$$

[20] 10. Compute $\int_S\int \text{curl}\,\mathbf{F}\cdot\mathbf{N}dS$ if S is the surface of the paraboloid $z=x^2+y^2$ with $x^2+y^2\le 9$. The normal points in the direction $\langle 0,0,1\rangle$ and $\mathbf{F}=y^2\mathbf{i}+x\mathbf{j}+z^2\mathbf{k}$.