Math 305 **Separable Equations**

This method is the most straightforward of all the methods. Basically, the ODE can be written as

$$y' = f(t)g(y).$$

As you can see the variables "separate" into the product of two different functions. From here you divide by $g(y)$, multiply by dt, and integrate both sides.

Example A. $y' = t^2 \cos y \qquad y(1) = 0$

1. Notice $\dfrac{dy}{dt} = t^2 \cos y$ so that $\dfrac{dy}{\cos y} = t^2 dt$.
2. Integrating $\displaystyle\int \sec y dy = \int t^2 dt$ so that $\ln|\sec y + \tan y| = \dfrac{t^3}{3} + c$
3. Plugging in IC: $\ln 1 = \dfrac{1^3}{3} + c$
4. Thus $\ln|\sec y + \tan y| = \dfrac{t^3}{3} - \dfrac{1}{3}$.

Example B. $y' = y^2 \sin t + \sin t + y^2 e^t + e^t \qquad y(0) = 1$

1. Rewrite the ODE as $\dfrac{dy}{dt} = (y^2+1)(\sin t + e^t)$.
2. Divide by y^2+1, multiply by dt to get $\dfrac{dy}{y^2+1} = (\sin t + e^t)dt$.
3. Integrating both sides we have $\displaystyle\int \frac{dy}{y^2+1} = \tan^{-1} y = \int (\sin t + e^t)dt = -\cos t + e^t + c$.
4. Plugging in IC: $\tan^{-1} 1 = -\cos 0 + e^0 + c$.
5. Thus $\tan^{-1} y = -\cos t + e^t + \dfrac{\pi}{4}$.

Hint I: As you can see, probably the hardest part of this problem is factoring the right-hand side!

Problems:

1. $y' = t^3 e^y + te^y + e^y \qquad y(1) = \ln 2$

2. $y' = ty + y \tan t + t + \tan t \qquad y(1) = 0$

3. $y' = t^2 - t \cos y - 1 + t + \cos y - t^2 \cos y \qquad y(2) = \dfrac{\pi}{4}$

4. $t' = e^{t+y} \qquad y(\ln 2) = \ln 3$

5. $y' = \dfrac{e^t + 1}{y^2 + 2} \qquad y(0) = -3$

Solutions:

1. $-e^{-y} = \dfrac{t^4}{4} + \dfrac{t^2}{2} + t - \dfrac{9}{4}$

2. $\ln(y+1) = \dfrac{t^2}{2} + \ln \sec t - \dfrac{1}{2}$

3. $-\cot y - \csc y = \dfrac{t^3}{3} + \dfrac{t^2}{2} - t - \sqrt{2} - \dfrac{11}{3}$

4. $e^y = -e^{-t} + \dfrac{7}{2}$

5. $\dfrac{y^3}{3} + 2y = e^t + t - 16$